

# Computing Preferred Extensions for Abstract Argumentation

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**Abstract**—In this document, we describe how to generate SAT instances through a general SAT-based abstract argumentation solver called MiniAF. We only focus on solving the reasoning tasks of preferred semantics on the ICCMA benchmarks.

## I. INTRODUCTION

SAT-based method is one of the most popular formal argumentation approaches for solving reasoning tasks of the abstract argumentation framework [1]. In this document, we use a general solver called MiniAF [6] to solve tasks from International Competition on Computational Models of Argumentation (ICCMA). For clearer understanding, we repeating the encoding method in this document.

## II. BASIC CONCEPTS

A given abstract argumentation framework (AF)  $AF = (A, R)$  can be represented by a directed graph, where  $A$  is a set of arguments and  $R \subseteq A \times A$  is the relation. For two arguments  $a, b \in A$ , the relation  $aRb$  means that  $a$  attacks  $b$  (which can be represented by  $a \rightarrow b$  as well), and we denote  $a^- = \{b | bRa\}$ . A set  $S \subseteq A$  defends an argument  $b \in A$  if for all  $a$  with  $aRb$  there is  $c \in S$  with  $cRa$ . Semantic  $\sigma$  represents a kind of property over a set of arguments, and a  $\sigma$ -extension  $E \subseteq A$  is a argument set with the property  $\sigma$ . There are definitions of some important semantic extensions.

- An extension  $E$  is *conflict-free* (*cf*) iff there are no arguments  $a, b \in E$  with  $aRb$ ;
- An extension  $E$  is *admissible* (*adm*) iff  $E$  is *cf* and  $E$  defends every  $a \in E$ ;
- An extension  $E$  is *complete* (*co*) iff  $E$  is *adm* and if  $E$  defends  $a$  then  $a \in E$ ;
- An extension  $E$  is *preferred* (*prf*) iff  $E$  is maximal *co*.

Given a semantic  $\sigma \in \{co, prf\}$  and an AF  $AF = (A, R)$ , an argument  $a \in A$  is *skeptically accepted* in  $AF$  if  $a$  is contained in every  $\sigma$ -extensions, is *credulously accepted* in  $AF$  if  $a$  is contained in some  $\sigma$ -extensions. There are some tasks base on a given AF  $AF = (A, R)$  and an argument  $a$ .

- EE- $\sigma$ : Enumerate all extensions  $E \subseteq A$  that are  $\sigma$ -extensions;

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- SE- $\sigma$ : Return an extension  $E \subseteq A$  that is a  $\sigma$ -extension;
- DC- $\sigma$ : Decide if  $a$  is credulously accepted under  $\sigma$ ;
- DS- $\sigma$ : Decide if  $a$  is skeptically accepted under  $\sigma$ .

## III. LABELLING ENCODING METHOD

This section introduces an equivalent way to define different types of semantics by labelling encoding method [4], [6]. Given a set of arguments  $A$ , a *labelling*  $L$  mapping each argument  $a \in A$  to  $\{in, out, undec\}$ , which means that  $a$  is accepted, rejected or the status is undecided, respectively. The set of all *labellings* for a given  $AF = (A, R)$  is denoted as  $\zeta(AF)$ .

$L \in \zeta(AF)$  is called a *complete labelling* (*co-L*) iff for any  $a \in A$  holds:

- $L(a) = in \Leftrightarrow \forall b \in a^-, L(b) = out$ ;
- $L(a) = out \Leftrightarrow \exists b \in a^-, L(b) = in$ .

A *co-L*  $L \in \zeta(AF)$  is equal to a *prf*-extension iff  $L$  maximize the set of arguments labelled *in*.

## IV. ENCODING FOR COMPLETE SEMANTICS

This section gives the classic encoding method for complete semantics [2] used in MiniAF [6]. Given an AF  $AF = (A, R)$  with  $|A| = k$ , and  $\Phi : \{1, \dots, k\} \rightarrow A$  is an indexing bijection.

At first, we define three symbols  $I_i, O_i, U_i$  for each argument  $a \in AF$  with indexing  $i$ , and for each argument  $a \in AF$ ,  $a$  can labelled exact one type label.

$$\bigwedge_{i \in \{1, \dots, k\}} ((I_i \vee O_i \vee U_i) \wedge (\neg I_i \vee \neg O_i) \wedge (\neg I_i \vee \neg U_i) \wedge (\neg O_i \vee \neg U_i)) \quad (1)$$

By the definition, for each argument  $a \in AF$  without any attackers,  $a$  should be labelled *in*.

$$\bigwedge_{\{i | \Phi(i)^- = \emptyset\}} (I_i \wedge O_i \wedge U_i) \quad (2)$$

Then, for each argument  $a \in AF$  with at least one attacker:  $L(a) = in \Rightarrow \forall b \in a^-, L(b) = out$ ;  $L(a) = in \Leftarrow \forall b \in a^-, L(b) = out$ .

$$\bigwedge_{\{i | \Phi(i)^- \neq \emptyset\}} \left( \bigwedge_{\{j | \Phi(j) \rightarrow \Phi(i)\}} \neg I_i \vee O_j \right) \quad (3)$$

$$\bigwedge_{\{i | \Phi(i)^- \neq \emptyset\}} \left( I_i \vee \left( \bigvee_{\{j | \Phi(j) \rightarrow \Phi(i)\}} \neg O_j \right) \right) \quad (4)$$

At last, for each argument  $a \in AF$  with at least one attacker:  
 $L(a) = out \Rightarrow \exists b \in a^-, L(b) = in$ ;  $L(a) = out \Leftarrow \exists b \in a^-, L(b) = in$ .

$$\bigwedge_{\{i|\Phi(i)^- \neq \emptyset\}} \left( \neg O_i \vee \left( \bigvee_{\{j|\Phi(j) \rightarrow \Phi(i)\}} \neg I_j \right) \right) \quad (5)$$

$$\bigwedge_{\{i|\Phi(i)^- \neq \emptyset\}} \left( \bigwedge_{\{j|\Phi(j) \rightarrow \Phi(i)\}} I_j \vee O_i \right) \quad (6)$$

All the above formulas (1)-(6) make up a conjunctive normal form (CNF)  $\Pi$ , which can be solved by a given SAT solver. At last, to enumerate all extensions, MiniAF excluding previous model  $s$  by add a formula  $\neg s$  to  $Pi$  after each time a model is found by the SAT solver, until the SAT solver return that there are no more model (UNSAT).

## V. PREFERRED SEMANTICS AND RELATED TASKS

MiniAF uses an improved PrefSAT algorithm [3] for computing *preferred labellings* (*prf-L*). The algorithm iterates over a set of *co-Ls* to identify the preferred ones and optimizes the process by set inclusion to maximise *co-Ls*.

To decide the credulous acceptance of an argument  $a$ , the CNF  $\Pi$  is updated to  $\Pi \wedge I_{\Phi^{-1}(a)}$ . To check the skeptically acceptance of an argument  $a$ , MiniAF subsequently enumerates all *prf-Ls* until it finds a labelling with  $L(a) \neq in$ .

## VI. BENCHMARK SELECTION

We use MiniAF to solve the tasks of *EE-prf*, *DS-prf* and *DC-prf* on the benchmarks from ICCMA-17, ICCMA-19 which can be downloaded from <http://argumentationcompetition.org/>. Following the definition of ‘interesting instance’ that one should not be solved by MiniSat [5] in a minute and should be solved by our own solver within 1 hour, We select some interesting instances from the intermediate results of MiniAF, which are in the format of “.cnf”.

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